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NsG-227

Final Report

Any process which will produce neutrinos is of great interest in stellar evolution. Since it has been possible to create a renormalizable intermediate vector boson theory, we have attempted to apply it to processes that have previously been inaccessible to ready computation. Thus, we have considered the neutrino form factor which leads to neutrino emission via the process[1] γ (plasmon) $\rightarrow \nu + \bar{\nu}$ and

$$\gamma + \gamma \rightarrow \nu + \bar{\nu}.$$

However, the latter can also be calculated on the basis of the conserved vector current if one of the photons is a coulomb photon, i.e.

$$\gamma + Z \rightarrow Z + \nu + \bar{\nu}$$

where Z is a nucleus of charge Z, assumed to be infinitely heavy. If the coulomb photon is replaced by a real photon, the matrix element vanishes[2]. This process was calculated by Matinyan and Tsilosani[3]. Their result is clearly incorrect since it is not gauge invariant. We have thus recalculated this process*.

By virtue of the conserved vector current hypothesis, and the current-current interaction hypothesis, there is a term in the weak interaction hamiltonian of the form

$$\mathcal{H}_W = G_F/\sqrt{2}\, \bar{\Psi}_\ell \gamma_\alpha (1 + \gamma_5) \Psi_{\nu_\ell} \bar{\Psi}_{\nu_\ell} \gamma_\alpha (1 + \gamma_5) \Psi_\ell$$

where Ψ_ℓ is the lepton field and Ψ_{ν_ℓ} is the corresponding neutrino field. If one applies Fierz transformation to this term,

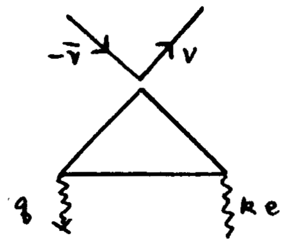
$$\mathcal{H}_W = G_F/\sqrt{2}\, \bar{\Psi}_\ell \gamma_\alpha (1 + \gamma_5) \Psi_\ell \bar{\Psi}_{\nu_\ell} \gamma_\alpha (1 + \gamma_5) \Psi_{\nu_\ell}$$

* After completion of this work, we discovered that this process was calculated correctly by L. Rosenberg, Phys. Rev. 129, 2786 (1963)

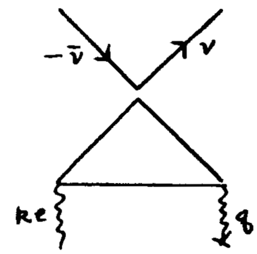
The lepton interacts with the electromagnetic field via the interaction

$$\mathcal{H}_{e.m.} = ie\,\bar{\psi}_\ell \gamma_\mu \psi_\ell A_\mu$$

Hence there are two diagrams



I



II

$$M_I = -e^2 \frac{G}{\sqrt{2}}\, \bar{u}^{(+)}_{\nu_\ell} \gamma_\alpha (1+\gamma_5) u^{(-)}_{-\bar{\nu}_\ell} \frac{1}{(2\pi)^4} \int d^4p \;\mathrm{Tr}\left\{ \gamma_\alpha (1+\gamma_5) \right.$$

$$\left. \times \frac{-i\gamma\cdot(p+q) + m_\ell}{(p+q)^2 + m_\ell^2}\, \gamma\cdot A\, \frac{-i\gamma\cdot p + m_\ell}{p^2 + m_\ell^2}\, \frac{\gamma\cdot e}{\sqrt{2k}}\, \frac{-i\gamma\cdot(p+k) + m_\ell}{(p+k)^2 + m_\ell^2} \right\}$$

$$M_{II} = -e^2 \frac{G}{\sqrt{2}}\, \bar{u}^{(+)}_{\nu_\ell} \gamma_\alpha (1+\gamma_5) u^{(-)}_{-\bar{\nu}_\ell} \frac{1}{(2\pi)^4} \int d^4p \;\mathrm{Tr}\left\{ \gamma_\alpha (1+\gamma_5) \right.$$

$$\left. \times \frac{-i\gamma\cdot(p-k) + m_\ell}{(p-k)^2 + m_\ell^2}\, \frac{\gamma\cdot e}{\sqrt{2k}}\, \frac{-i\gamma\cdot p + m_\ell}{p^2 + m_\ell^2}\, \gamma\cdot A\, \frac{-i\gamma\cdot(p-q) + m_\ell}{(p-q)^2 + m_\ell^2} \right\}$$

$$M = M_I + M_{II}$$

Let $\qquad \ell_\alpha = \bar{u}^{(+)}_{\nu_\ell} \gamma_\alpha (1+\gamma_5) u^{(-)}_{-\bar{\nu}_\ell} \qquad$ Then,

$$M = \sqrt{2}\, e^2 G_F\, \ell_\alpha \frac{1}{(2\pi)^4} \int d^4p \;\mathrm{Tr}\left[\gamma_5 \gamma_\alpha \frac{-i\gamma\cdot(p-k) + m_\ell}{(p-k)^2 + m_\ell^2} \frac{\gamma\cdot e}{\sqrt{2k}} \right.$$

$$\left. \times \frac{-i\gamma\cdot p + m_\ell}{p^2 + m_\ell^2}\, \gamma\cdot A\, \frac{-i\gamma\cdot(p-q) + m_\ell}{(p-q)^2 + m_\ell^2} \right]$$

$$\equiv \sqrt{2}\, e^2 G_F\, \ell_\alpha\, e_\mu A_\nu H^\alpha_{\mu\nu}$$

and

$$H^{\alpha}_{\mu\nu} = \frac{1}{(2\pi)^4} \int d^4p \; \mathrm{Tr}\left[\gamma_5 \gamma_\alpha \frac{-i\gamma\cdot(p-k)+m_e}{(p-k)^2+m_e^2} \gamma_\mu \frac{-i\gamma\cdot p+m_e}{p^2+m_e^2} \gamma_\nu \frac{-i\gamma\cdot(p-q)+m_e}{(p-q)^2+m_e^2}\right]$$

Gauge invariance imposes the conditions

$$k_\mu H^{\alpha}_{\mu\nu} = q_\nu H^{\alpha}_{\mu\nu} = 0$$

We can write $H^{\alpha}_{\mu\nu}$ in terms of the following eight form factors:

$$\begin{aligned} H^{\alpha}_{\mu\nu} = \; & F_1 \, \varepsilon_{\alpha\mu\nu\sigma} k_\sigma + F_2 \, \varepsilon_{\alpha\mu\nu\sigma} q_\sigma \\ & + F_3 \, \varepsilon_{\mu\nu\rho\lambda} k_\rho q_\lambda k_\alpha + F_4 \, \varepsilon_{\mu\nu\rho\lambda} k_\rho q_\lambda q_\alpha \\ & + F_5 \, \varepsilon_{\alpha\mu\rho\lambda} k_\rho q_\lambda k_\nu + F_6 \, \varepsilon_{\alpha\mu\rho\lambda} k_\rho q_\lambda q_\nu \\ & + F_7 \, \varepsilon_{\alpha\nu\rho\lambda} k_\rho q_\lambda k_\mu + F_8 \, \varepsilon_{\alpha\nu\rho\lambda} k_\rho q_\lambda q_\mu \end{aligned}$$

As will be seen, F_1 and F_2 are infinite and require a renormalization; all other form factors are finite. However, because of the requirement of gauge invariance, we shall see that F_1 and F_2 can be eliminated in terms of the others and thus, we shall arrive at a finite, gauge invariant, renormalized result.

$$\begin{aligned} k_\mu H^{\alpha}_{\mu\nu} = \; & F_2 \, \varepsilon_{\alpha\mu\nu\sigma} k_\mu q_\sigma + F_7 \, \varepsilon_{\alpha\nu\rho\lambda} k_\rho q_\lambda k^2 \\ & + F_8 \, \varepsilon_{\alpha\nu\rho\lambda} k_\rho q_\lambda \, k\cdot q \\ = \; & 0 \end{aligned}$$

Since $k^2 = 0$,

$$-F_2 \, \varepsilon_{\alpha\nu\rho\lambda} k_\rho q_\lambda + F_8 \, k\cdot q \, \varepsilon_{\alpha\nu\rho\lambda} k_\rho q_\lambda = 0$$

$$F_2 = k\cdot q \, F_8$$

$$q_\nu H^\alpha_{\mu\nu\lambda} = F_1 \varepsilon_{\alpha\mu\nu\sigma} q_\nu k_\sigma + F_5 \varepsilon_{\alpha\mu\rho\lambda} k_\rho q_\lambda k\cdot q$$

$$+ F_6 \varepsilon_{\alpha\mu\rho\lambda} k_\rho q_\lambda q^2$$

$$F_1 = q^2 F_6 + k\cdot q F_5$$

Thus, we replace F_2 by $k\cdot q\, F_8$ and F_1 by $q^2F_6 + k\cdot q\, F_5$. We must now compute F_3, F_4, F_5, F_6, F_7, F_8. As we shall see, F_3 and F_4 never appear.

$$H^\alpha_{\mu\nu} = \frac{2}{(2\pi)^4} \int dx_1 dx_2 dx_3\ \delta(x_1+x_2+x_3-1) \int d^4p$$

$$\times \frac{Tr\left[\gamma_5 \gamma_\alpha \{-i\gamma\cdot(p-k)+m_\ell\} \gamma_\mu \{-i\gamma\cdot p+m_\ell\} \gamma_\nu \{-i\gamma\cdot(p-q)+m_\ell\}\right]}{[p^2 - 2p\cdot(kx_1+qx_2) + q^2x_2 + m_\ell^2]^3}$$

Performing the indicated operations

$$F_5 = \frac{16i}{(2\pi)^4} \int dx_1 dx_2 dx_3\ \delta(x_1+x_2+x_3-1) \int d^4p \frac{x_1 x_2}{[p^2-(kx_1+qx_2)^2+q^2x_2+m_\ell^2]^3}$$

$$= F_8$$

$$F_6 = \frac{16i}{(2\pi)^4} \int dx_1 dx_2 dx_3\ \delta(x_1+x_2+x_3-1) \int d^4p \frac{x_2(x_2-1)}{[p^2-(kx_1+qx_2)^2+q^2x_2+m_\ell^2]^3}$$

$$F_7 = \frac{16i}{(2\pi)^4} \int dx_1 dx_2 dx_3\ \delta(x_1+x_2+x_3-1) \int d^4p \frac{x_1(x_1-1)}{[p^2-(kx_1+qx_2)^2+q^2x_2+m_\ell^2]^3}$$

Now,

$$H^{\alpha}_{\mu\nu} = F_1\ \varepsilon_{\alpha\mu\nu\sigma}\, k_\sigma + F_2\ \varepsilon_{\alpha\mu\nu\sigma}\, q_\sigma$$
$$+ F_5\ \varepsilon_{\alpha\mu\rho\lambda}\, k_\rho q_\lambda k_\nu + F_6\ \varepsilon_{\alpha\mu\rho\lambda}\, k_\rho q_\lambda q_\nu$$
$$+ F_7\ \varepsilon_{\alpha\nu\rho\lambda}\, k_\rho q_\lambda k_\mu + F_8\ \varepsilon_{\alpha\nu\rho\lambda}\, k_\rho q_\lambda q_\mu$$

$$H^{\alpha}_{\mu\nu}\, \ell_\alpha e_\mu A_\nu = F_1\ \varepsilon_{\alpha\mu\nu\sigma}\, \ell_\alpha e_\mu A_\nu k_\sigma + F_2\ \varepsilon_{\alpha\mu\nu\sigma}\, \ell_\alpha e_\mu A_\nu q_\sigma$$
$$+ F_5\ k\cdot A\ \varepsilon_{\alpha\mu\rho\lambda}\, \ell_\alpha e_\mu k_\rho q_\lambda + F_6\ q\cdot A\ \varepsilon_{\alpha\mu\rho\lambda}\, \ell_\alpha e_\mu k_\rho q_\lambda$$
$$+ F_7\ k\cdot e\ \varepsilon_{\alpha\nu\rho\lambda}\, \ell_\alpha A_\nu k_\rho q_\lambda + F_8\ q\cdot e\ \varepsilon_{\alpha\nu\rho\lambda}\, \ell_\alpha A_\nu k_\rho q_\lambda$$

But $$k\cdot e = q\cdot A = 0$$

Thus,

$$H^{\alpha}_{\mu\nu}\, \ell_\alpha e_\mu A_\nu = F_1\ \varepsilon_{\alpha\mu\nu\sigma}\, \ell_\alpha e_\mu A_\nu k_\sigma + F_2\, \varepsilon_{\alpha\mu\nu\sigma}\, \ell_\alpha e_\mu A_\nu q_\sigma$$
$$+ F_5\ k\cdot A\ \varepsilon_{\alpha\mu\rho\lambda}\, \ell_\alpha e_\mu k_\rho q_\lambda + F_8\ q\cdot e\ \varepsilon_{\alpha\nu\rho\lambda}\, \ell_\alpha A_\nu k_\rho q_\lambda$$

Now,

$$k\cdot A\ \varepsilon_{\alpha\mu\rho\lambda}\, \ell_\alpha e_\mu k_\rho q_\lambda = -k\cdot q\ \varepsilon_{\alpha\mu\rho\lambda}\, A_\alpha \ell_\mu e_\rho k_\lambda$$
$$- k\cdot \ell\ \varepsilon_{\alpha\mu\rho\lambda}\, e_\alpha k_\mu q_\rho A_\lambda$$
$$= -k\cdot q\ \varepsilon_{\alpha\mu\nu\sigma}\, \ell_\alpha e_\mu A_\nu k_\sigma$$
$$- k\cdot \ell\ \varepsilon_{\alpha\mu\rho\lambda}\, e_\alpha k_\mu q_\rho A_\lambda$$

$$q\cdot e\ \varepsilon_{\alpha\mu\rho\lambda}\, e_\alpha k_\mu q_\rho A_\lambda = -q\cdot \ell\ \varepsilon_{\alpha\nu\rho\lambda}\, A_\alpha k_\nu q_\lambda e_\lambda$$
$$- q\cdot k\ \varepsilon_{\alpha\nu\rho\lambda}\, \ell_\alpha e_\nu A_\rho q_\lambda + q^2\ \varepsilon_{\alpha\mu\nu\sigma}\, \ell_\alpha e_\mu A_\nu k_\sigma$$

Making use of

$$F_2 = k\cdot q\, F_8$$

$$F_1 = q^2 F_6 + k\cdot q\, F_5$$

$$H^{\alpha}_{\mu\nu}\, l_\alpha e_\mu A_\nu = (q^2 F_6 + k\cdot q\, F_5)\, \varepsilon_{\alpha\mu\nu\sigma}\, l_\alpha e_\mu A_\nu k_\sigma$$

$$+ k\cdot q\, F_8\, \varepsilon_{\alpha\mu\nu\sigma}\, l_\alpha e_\mu A_\nu q_\sigma + F_5 (- k\cdot q\, \varepsilon_{\alpha\mu\nu\sigma}\, l_\alpha e_\mu A_\nu k_\sigma$$

$$- k\cdot l\, \varepsilon_{\alpha\mu\rho\lambda}\, e_\alpha k_\mu q_\rho A_\lambda) + F_8 (- q\cdot l\, \varepsilon_{\alpha\nu\rho\lambda}\, A_\alpha k_\nu q_\rho e_\lambda$$

$$- q\cdot k\, \varepsilon_{\alpha\mu\nu\sigma}\, l_\alpha e_\mu A_\nu q_\sigma + q^2\, \varepsilon_{\alpha\mu\nu\sigma}\, l_\alpha e_\mu A_\nu k_\sigma)$$

$$= q^2 (F_6 + F_8)\, \varepsilon_{\alpha\mu\nu\sigma}\, l_\alpha e_\mu A_\nu k_\sigma - k\cdot l\, F_5\, \varepsilon_{\alpha\mu\nu\sigma}\, e_\alpha k_\mu q_\nu A_\sigma$$

$$- q\, l\, F_8\, \varepsilon_{\alpha\nu\rho\lambda}\, A_\alpha k_\nu q_\rho e_\lambda$$

$$= q^2 (F_6 + F_8)\, \varepsilon_{\alpha\mu\nu\sigma}\, l_\alpha e_\mu A_\nu k_\sigma - (k\cdot l\, F_5 - l\cdot q\, F_8)$$

$$\times\, \varepsilon_{\alpha\mu\rho\lambda}\, e_\alpha k_\mu q_\rho A_\lambda$$

$$= q^2 (F_6 + F_8)\, \varepsilon_{\alpha\mu\nu\sigma}\, l_\alpha e_\mu A_\nu k_\sigma$$

$$- l\cdot (k - q)\, F_5\, \varepsilon_{\alpha\mu\rho\lambda}\, l_\alpha k_\mu q_\rho A_\lambda$$

since $F_5 = F_8$. But $k - q = \nu + \bar{\nu}$

and thus $\quad l\cdot (k-q) = l\cdot(\nu + \bar{\nu}) = 0.$ Hence,

$$H^{\alpha}_{\mu\nu}\, l_\alpha e_\mu A_\nu = q^2 (F_6 + F_8)\, \varepsilon_{\alpha\mu\nu\sigma}\, l_\alpha e_\mu A_\nu k_\sigma$$

$$= q^2 (F_6 + F_8)\, A_4\, \varepsilon_{4\alpha\mu\sigma}\, l_\alpha e_\mu k_\sigma$$

$$= q^2 (F_6 + F_8)\, A_4\, \vec{l}\cdot \vec{e}\times\vec{k}$$

Let $$F = F_6 + F_8$$

Thus

$$H = \sqrt{2}\, e^2\, G_F\, F\, q^2 A_4\quad \vec{\ell}\cdot\frac{\vec{e}}{\sqrt{2k}}\times\vec{k}$$

$$q^2 A_4 = i Z \qquad ; \qquad \text{thus,}$$

$$H = i\sqrt{2}\, Z e^2\, G_F\, F(\vec{k},\vec{q})\quad \frac{\vec{\ell}\cdot\vec{e}\times\vec{k}}{\sqrt{2k}}$$

To obtain a cross-section we need

$$\sum_{\substack{e\\ s\bar{s}}} |H|^2 = 2 Z^2 e^4 G_F^2 |F(\vec{k},\vec{q})|^2 \frac{1}{2k} \sum_{\substack{e\\ s\bar{s}}} (\vec{\ell}^{\,+}\cdot\vec{e}\times\vec{k})(\vec{\ell}\cdot\vec{e}\times\vec{k})$$

$$= 2 Z^2 e^4 G_F^2 |F(\vec{k},\vec{q})|^2 \frac{1}{2k} \sum_{e,s,\bar{s}} (\vec{\ell}^{\,+}\times\vec{k}\cdot\vec{e})(\vec{\ell}\times\vec{k}\cdot\vec{e})$$

$$= \frac{Z^2 e^4}{k} G_F^2 |F(\vec{k},\vec{q})|^2 \sum_{e,s,\bar{s}} (\vec{\ell}^{\,+}\times\vec{k})\cdot(\vec{\ell}\times\vec{k})$$

$$= \frac{Z^2 e^4}{k} G_F^2 |F(\vec{k},\vec{q})|^2 \sum_{s\bar{s}} (\vec{\ell}^{\,+}\cdot\vec{\ell}\, k^2 - \vec{\ell}^{\,+}\cdot\vec{k}\,\vec{\ell}\cdot\vec{k})$$

$$= \frac{Z^2 e^4}{k} G_F^2 |F(\vec{k},\vec{q})|^2 \sum_{s\bar{s}} \ell_i^+ \ell_j (k^2 \delta_{ij} - k_i k_j)$$

Now,

$$\sum_{s\bar{s}} \ell_i^+ \ell_j = - \sum_{s\bar{s}} \bar{u}^{(-)}_{-\bar{\nu}} \gamma_i (1+\gamma_5) u^{(+)}_{\nu}\, \bar{u}^{(+)}_{\nu} \gamma_j (1+\gamma_5) u^{(-)}_{-\bar{\nu}}$$

$$= - \frac{Tr[-i\gamma\cdot\bar{\nu}\,\gamma_i (1+\gamma_5) - i\gamma\cdot\nu\, \gamma_j (1+\gamma_5)]}{4\nu\bar{\nu}}$$

$$= \frac{1}{2\nu\bar{\nu}} Tr[\gamma\cdot\bar{\nu}\,\gamma_i\,\gamma\cdot\nu\,\gamma_j (1+\gamma_5)]$$

$$= \frac{2}{\nu\bar{\nu}} \left\{ \bar{\nu}_i \nu_j + \bar{\nu}_j \nu_i - \nu\bar{\nu}\,\delta_{ij} + \varepsilon_{\alpha i\beta j}\bar{\nu}_\alpha \nu_\beta \right\}$$

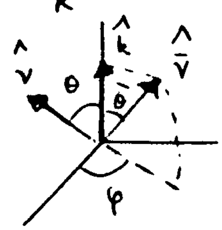
Thus,

$$\sum |M|^2 = \frac{Z^2 e^4}{k} G_F^2 \, |F(\vec{k},\vec{q})|^2 \, \frac{2}{\nu\bar{\nu}} \left\{ k^2 \left(2\vec{\bar{\nu}}\cdot\vec{\nu} - 3\nu\cdot\bar{\nu} \right) - 2\vec{k}\cdot\vec{\nu}\,\vec{k}\cdot\vec{\bar{\nu}} + k^2 \nu\cdot\bar{\nu} \right\}$$

$$= \frac{Z^2 e^4}{k} G_F^2 \, |F(\vec{k},\vec{q})|^2 \, \frac{4}{\nu\bar{\nu}} \left\{ k^2 \vec{\nu}\cdot\vec{\bar{\nu}} - k^2 \nu\cdot\bar{\nu} - \vec{k}\cdot\vec{\nu}\,\vec{k}\cdot\vec{\bar{\nu}} \right\}$$

$$= \frac{Z^2 e^4}{k} G_F^2 \, |F(\vec{k},\vec{q})|^2 \, \frac{4}{\nu\bar{\nu}} \left\{ k^2 \nu\bar{\nu} - \vec{k}\cdot\vec{\nu}\,\vec{k}\cdot\vec{\bar{\nu}} \right\}$$

$$= 4 \frac{Z^2 e^4}{k} G_F^2 \, |F(\vec{k},\vec{q})|^2 \, k^2 \left(1 - \cos\theta \cos\bar{\theta} \right)$$



Hence,

$$\frac{d^3\sigma}{d\nu\, d\Omega\, d\bar{\Omega}} = 2\pi \sum |M|^2 \frac{\nu^2 (k-\nu)^2}{(2\pi)^6}$$

$$\frac{d^3\sigma}{d\nu\, d\Omega\, d\bar{\Omega}} = \frac{4k}{(2\pi)^5} Z^2 e^4 G_F^2 \, |F(\vec{k},\vec{q})|^2 (1-\cos\theta\cos\bar{\theta})\, \nu^2 (k-\nu)^2$$

$$= \frac{2}{\pi^3} Z^2 \alpha^2 G_F^2 \, |F(\vec{k},\vec{q})|^2 (1-\cos\theta\cos\bar{\theta})\, k\, \nu^2 (k-\nu)^2$$

$$F(\vec{k},\vec{q}) = \frac{16 i}{(2\pi)^4} \int dx_1 dx_2 dx_3 \, \delta(x_1+x_2+x_3-1) \int d^4p \, \frac{x_2(x_1+x_2-1)}{[p^2 - (kx_1 + qx_2)^2 + q^2 x_2 + m_e^2]^3}$$

$$= -\frac{16}{(2\pi)^4} \frac{\pi^2}{2} \int dx_1 dx_2 dx_3 \, \delta(x_1+x_2+x_3-1) \, \frac{x_2(x_1+x_2-1)}{-(kx_1+qx_2)^2 + q^2 x_2 + m_e^2}$$

$$= \frac{1}{2\pi^2} \int dx_1 dx_2 dx_3 \, \delta(x_1+x_2+x_3-1) \, \frac{x_2 x_3}{q^2 x_2(1-x_2) - 2k\cdot q\, x_1 x_2 + m_e^2}$$

$$= \frac{1}{2\pi^2} \int dx_1 dx_2 dx_3 \; \delta(x_1+x_2+x_3-1) \; \frac{x_2 x_3}{(q^2-2k\cdot q)x_2(1-x_2)+2k\cdot q\, x_2x_3 + m_\ell^2}$$

$$= \frac{1}{2\pi^2} \int dx_1 dx_2 dx_3 \; \delta(x_1+x_2+x_3-1) \; \frac{x_2 x_3}{(k-q)^2 x_2(1-x_2) + 2k\cdot q\, x_2x_3 + m_\ell^2}$$

$$= \frac{1}{2\pi^2} \int_0^1 dx_2 \int_0^{1-x_2} dx_3 \; \frac{x_2 x_3}{(k-q)^2 x_2(1-x_2) + 2k\cdot q\, x_2x_3 + m_\ell^2}$$

$$= \frac{1}{2\pi^2} \; \frac{1}{2k\cdot q} \left\{ \frac{1}{2} - \int_0^1 dx \; \frac{(k-q)^2 x(1-x) + m_\ell^2}{2kqx} \right.$$

$$\left. \times \log\left[1 + \frac{2k\cdot q\; x(1-x)}{(k-q)^2 x(1-x) + m_\ell^2} \right] \right\}$$

$$= \frac{1}{4\pi^2} \; \frac{1}{2k\cdot q} \left\{ 1 - \int_0^1 dx \; \frac{(k-q)^2 x(1-x) + m_\ell^2}{2k\cdot q\; x(1-x)} \right.$$

$$\left. \times \log\left[1 + \frac{2k\cdot q\; x(1-x)}{(k-q)^2 x(1-x) + m_\ell^2} \right] \right\}$$

Let $\quad x = \frac{1}{2}(1-y)$

$$F(\vec{k},\vec{q}) = \frac{1}{4\pi^2} \; \frac{1}{2k\cdot q} \left\{ 1 - \frac{1}{2} \int_{-1}^1 dy \; \frac{(k-q)^2 (1-y^2) + 4m_\ell^2}{2k\cdot q\,(1-y^2)} \right.$$

$$\left. \times \log\left[1 + \frac{2k\cdot q\,(1-y^2)}{(k-q)^2(1-y^2) + 4m_\ell^2} \right] \right\}$$

$$= \frac{1}{4\pi^2}\ \frac{1}{2k\cdot q}\left\{ 1 - \int_0^1 dy\ \frac{(k-q)^2(1-y^2)+4m_\ell^2}{2k\cdot q\,(1-y^2)}\right.$$

$$\left.\times \log\left[1 + \frac{2k\cdot q\ (1-y^2)}{(k-q)^2(1-y^2)+4m_\ell^2}\right]\right\}$$

If $k \ll m_\ell$,

$$F(\vec{k},\vec{q}) \cong \frac{1}{4\pi^2}\ \frac{1}{2k\cdot q}\left\{ 1 - \int_0^1 dy\ \frac{(k-q)^2(1-y^2)+4m_\ell^2}{2k\cdot q\,(1-y^2)}\right.$$

$$\times\left[\frac{2k\cdot q\ (1-y^2)}{(k-q)^2(1-y^2)+4m_\ell^2} - \frac{1}{2}\left(\frac{2k\cdot q\ (1-y^2)}{(k-q)^2(1-y^2)+4m_\ell^2}\right)^2\right.$$

$$\left.\left.+\frac{1}{3}\left(\frac{2k\cdot q\,(1-y^2)}{(k-q)^2(1-y^2)+4m_\ell^2}\right)^3\right]\right\}$$

$$\cong \frac{1}{4\pi^2}\ \frac{1}{2k\cdot q}\left\{\frac{1}{2}\int_0^1 dy\ \frac{2k\cdot q\ (1-y^2)}{(k-q)^2(1-y^2)+4m_\ell^2}\right.$$

$$\left.-\frac{1}{3}\int_0^1 dy\left(\frac{2k\cdot q\ (1-y^2)}{(k-q)^2(1-y^2)+4m_\ell^2}\right)^2\right\}$$

$$\cong \frac{1}{8\pi^2}\int_0^1 dy\ \frac{1-y^2}{4m_\ell^2+(k-q)^2(1-y^2)} - \frac{1}{12\pi^2}\int_0^1 dy\ \frac{2k\cdot q\ (1-y^2)^2}{[4m_\ell^2+(k-q)^2(1-y^2)]^2}$$

$$\cong \frac{1}{8\pi^2}\ \frac{1}{4m_\ell^2}\int_0^1 dy\ (1-y^2)\left[1 - \frac{(k-q)^2}{4m_\ell^2}(1-y^2)\right]$$

$$-\frac{1}{12\pi^2}\int_0^1 dy\ \frac{2k\cdot q}{16m_\ell^4}(1-y^2)^2$$

$$\cong \frac{1}{8\pi^2}\frac{1}{4m_\ell^2}\left\{\frac{2}{3} - \frac{(k-q)^2}{4m_\ell^2}\frac{8}{15}\right\} - \frac{1}{12\pi^2}\frac{2k\cdot q}{(4m_\ell^2)^2}\frac{8}{15}$$

$$\cong \frac{1}{8\pi^2}\frac{1}{4m_\ell^2}\left\{\frac{2}{3} - \frac{8}{15}\frac{(k-q)^2}{4m_\ell^2} - \frac{2}{3}\cdot\frac{8}{15}\frac{2k\cdot q}{4m_\ell^2}\right\}$$

$$\cong \frac{1}{48\pi^2}\frac{1}{m_\ell^2}\left\{1 - \frac{1}{5}\frac{(\nu+\bar\nu)^2}{m_\ell^2} - \frac{8}{15}\frac{2k\cdot q}{4m_\ell^2}\right\}$$

$$\frac{d^3\sigma}{d\nu\, d\Omega\, d\bar\Omega} \cong \frac{1}{24\pi^5}\frac{Z^2\alpha^2 G_F^2}{m_\ell^2} k\,(1-\cos\theta\cos\bar\theta)\,\nu^2(k-\nu)^2$$

$$\times\left\{1 + \frac{2}{5}\frac{\nu(k-\nu)}{m_\ell^2}(1-\cos\theta_{\nu\bar\nu}) - \frac{4}{15}\frac{\vec k\cdot\vec q}{m_\ell^2}\right\}$$

$$\cos\theta_{\nu\bar\nu} = \sin\theta\sin\bar\theta\cos\varphi + \cos\theta\cos\bar\theta$$

$$\vec k\cdot\vec q = k\,[k - \nu\cos\theta - \bar\nu\cos\bar\theta] = k\,[k - \nu\cos\theta - (k-\nu)\cos\bar\theta]$$

$$\frac{d^3\sigma}{d\nu\, d\Omega\, d\cos\bar\theta} = \frac{1}{12\pi^4}\frac{Z^2\alpha^2 G_F^2}{m_\ell^2} k\,(1-\cos\theta\cos\bar\theta)\,\nu^2(k-\nu)^2$$

$$\times\left\{1 + \frac{2}{5}\frac{\nu(k-\nu)}{m_\ell^2}(1-\cos\theta\cos\bar\theta) - \frac{4}{15}\frac{k}{m_\ell^2}(k-\nu\cos\theta-(k-\nu)\cos\bar\theta)\right\}$$

$$\frac{d^3\sigma}{d\nu\, d\cos\theta\, d\cos\bar\theta} \cong \frac{1}{6\pi^3}\frac{Z^2\alpha^2 G_F^2}{m_\ell^2} k\,\nu^2(k-\nu)^2\,(1-\cos\theta\cos\bar\theta)$$

$$\times\left\{1 + \frac{2}{5}\frac{\nu(k-\nu)}{m_\ell^2}(1-\cos\theta\cos\bar\theta) - \frac{4}{15}\frac{k}{m_\ell^2}\left[k-\nu\cos\theta-(k-\nu)\cos\bar\theta\right]\right\}$$

$$\frac{d\sigma}{d\nu} \cong \frac{4}{6\pi^3} \frac{Z^2 \alpha^2 G_F^2}{m_\ell^2} k \nu^2 (k-\nu)^2 \left\{ 1 + \frac{2}{5}\cdot\frac{10}{9} \frac{\nu(k-\nu)}{m_\ell^2} - \frac{4}{15}\frac{k^2}{m_\ell^2} \right\}$$

$$\sigma \cong \frac{2}{3\pi^3} \frac{Z^2 \alpha^2 G_F^2}{m_\ell^2} \frac{k^6}{30} \left\{ \left(1 - \frac{4}{15}\frac{k^2}{m_\ell^2}\right) + \frac{4}{42}\frac{k^2}{m_\ell^2} \right\}$$

$$\cong \frac{1}{45\pi^3} \frac{Z^2 \alpha^2 G_F^2}{m_\ell^2} k^6 \left\{ 1 - \frac{6}{35}\frac{k^2}{m_\ell^2} \right\}$$

This value of σ is very good for muonneutrinos for all values of k of interest in astrophysics since KT ~ 1 Mev in the problems of interest and m_ℓ = 104 Mev. But for electron neutrinos this is not correct.

The luminosity in this case is

$$L = N \overline{Z^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{k/KT} - 1} k \, \sigma_0(k)$$

where N is average number density of elements in the star and $\overline{Z}^2$ is the average charge of the elements, i.e.

$$N \overline{Z^2} = \sum_i N_i Z_i^2$$

and $\sigma_0(k) \equiv \dfrac{\sigma(k)}{Z^2}$ Then,

$$\sigma_0(k) \cong \frac{1}{45\pi^3} \alpha^2 G_F^2 m_\ell^2 \left(\frac{k}{m_\ell}\right)^6 \left\{ 1 - \frac{6}{35}\frac{k^2}{m_\ell^2} \right\}$$

$$L = N \overline{Z^2} \frac{1}{(2\pi)^3} \frac{1}{45\pi^3} 4\pi \, \alpha^2 G_F^2 m_\ell^2$$

$$\times \int_0^\infty dk \frac{k^3}{e^{k/KT} - 1} \left(\frac{k}{m_\ell}\right)^6 \left\{ 1 - \frac{6}{35}\frac{k^2}{m_\ell^2} \right\}$$

Thus,

$$L = \frac{N\overline{Z^2}}{90\pi^5} \quad \alpha^2 G_P^2 m_\ell^2 \left\{ \frac{(kT)^{10}}{m_\ell^6} \int_0^\infty dx \; \frac{x^9}{e^x - 1} \right.$$

$$\left. - \frac{6}{35} \frac{(kT)^{12}}{m_\ell^8} \int_0^\infty dx \; \frac{x^{11}}{e^x - 1} \right\}$$

$$= N\overline{Z^2} \quad \frac{\alpha^2 G_F^2 m_\ell^2}{90\pi^5} \left\{ \frac{5}{66} \frac{(kT)^{10}}{m_\ell^6} \frac{(2\pi)^{10}}{20} \right.$$

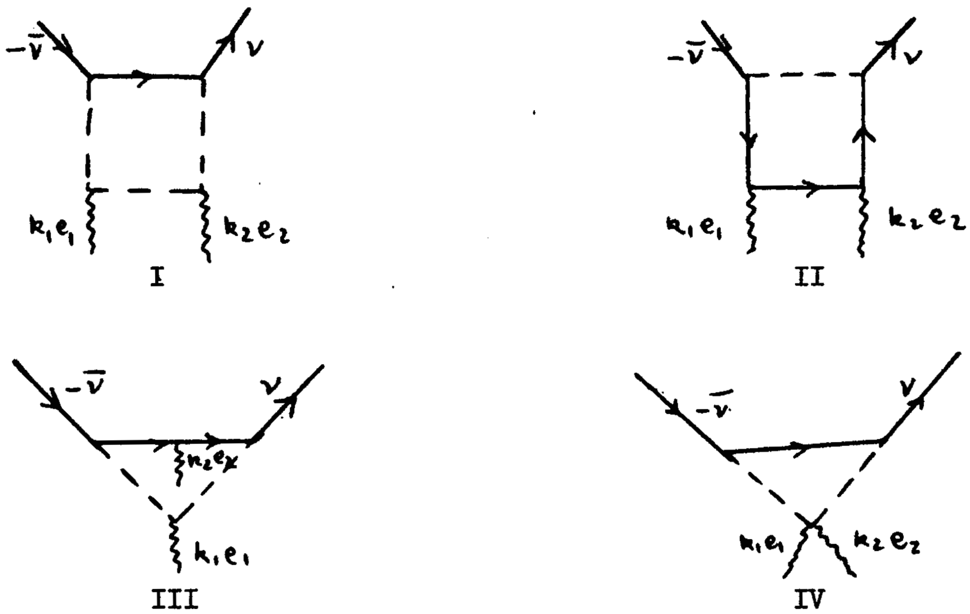
$$\left. - \frac{6}{35} \frac{(kT)^{12}}{m_\ell^8} \frac{691}{2730} \frac{(2\pi)^{12}}{24} \right\}$$

$$= N\overline{Z^2} \quad \frac{\alpha^2 G_F^2 m_\ell^2}{90\pi^5} \quad \frac{(kT)^{10}}{m_\ell^6} \quad \frac{(2\pi)^{10}}{4\cdot 66} \left\{ 1 - \left(\frac{kT}{m_\ell}\right)^2 \frac{(2\pi)^2 \cdot 11 \cdot 691}{35\cdot 455} \right\}$$

$$= N\overline{Z^2} \quad \alpha^2 G_F^2 m_\ell^2 \quad \frac{(kT)^{10}}{m_\ell^6} \quad \frac{64\pi^5}{1485} \left\{ 1 - \left(\frac{kT}{m_\ell}\right)^2 \frac{2\pi^2 \cdot 11 \cdot 691}{35\cdot 455} \right\}.$$

For other values of k, F (k,q) is being evaluated on a computer and the neutrino luminosity computed as well.

Matinyan and Tsilosani estimated the process $\gamma + \gamma \to \nu + \bar{\nu}$ making use of the intermediate vector boson. That result is also clearly not gauge invariant and, thus, incorrect. We have started the evaluation of this process. The diagrams which contribute are



and the same diagrams with $k_1\ e_1$ and $k_2\ e_2$ interchanged. The last diagram vanishes. Thus, we need only consider diagrams I, II, III.

$$M = M_{I} + M_{II} + M_{III} + (k_1 e_1 \leftrightarrow k_2 e_2)$$

$$M_{I} = -g^2 e^2 \frac{1}{\sqrt{2k_1}} \frac{1}{\sqrt{2k_2}} \bar{u}^{(+)}_{\nu} \gamma_\alpha (1+\gamma_5) \frac{1}{(2\pi)^4} \int d^4k$$

$$\times \frac{-i\gamma\cdot(\nu-k-k_2)+m_\ell}{(\nu-k-k_2)^2+m_\ell^2} \gamma_\alpha (1+\gamma_5) u^{(-)}_{-\bar{\nu}} \frac{1}{(k-k_1)^2+m^2} \frac{1}{k^2+m^2} \frac{1}{(k+k_2)^2+m^2}$$

$$\times (2k-k_1)\cdot e_1 \ (2k+k_2)\cdot e_2$$

$$M_{II} = g^2 e^2 \frac{1}{\sqrt{2k_1}} \frac{1}{\sqrt{2k_2}} \bar{u}^{(+)}_{\nu} \gamma_\alpha (1+\gamma_5) \frac{1}{(2\pi)^4} \int d^4k$$

$$\times \frac{-i\gamma\cdot(\nu-k)+m_\ell}{(\nu-k)^2+m_\ell^2} \gamma\cdot e_2 \frac{-\frac{i}{2}\gamma\cdot(\nu-\bar{\nu}+k_2-k_1-2k)+m_\ell}{\frac{1}{4}(\nu-\bar{\nu}+k_2-k_1-2k)^2+m_\ell^2} \gamma\cdot e_1 \frac{i\gamma\cdot(\bar{\nu}+k)+m_\ell}{(\bar{\nu}+k)^2+m_\ell^2}$$

$$\times \gamma_\alpha (1+\gamma_5) u^{(-)}_{-\bar{\nu}} \frac{1}{k^2+m^2}$$

$$M_{III} = g^2 e^2 \frac{1}{\sqrt{2k_1}} \frac{1}{\sqrt{2k_2}} \bar{u}_\nu^{(+)} \gamma_\alpha (1+\gamma_5) \frac{1}{(2\pi)^4} \int d^4 p$$

$$\times \frac{-i\gamma\cdot(\nu - p - \frac{k_2}{2}) + m_\ell}{(\nu - p - \frac{k_2}{2})^2 + m_\ell^2} \gamma\cdot e_1 \frac{-i\gamma\cdot(\frac{k_2}{2} - \bar{\nu} - p) + m_\ell}{(\frac{k_2}{2} - \bar{\nu} - p)^2 + m_\ell^2} \gamma_\alpha (1+\gamma_5) u_{-\bar{\nu}}^{(-)}$$

$$\times \frac{1}{(p + \frac{k_2}{2})^2 + m^2} \, 2p\cdot e_2 \, \frac{1}{(p - \frac{k_2}{2})^2 + m^2}$$

Note that all matrix elements are finite if integrated symmetrically since then the most singular piece of the integral behaves like

$$\int \frac{d^4 p}{p^8} p^2 = \int \frac{p^5}{p^8} dp = \int \frac{dp}{p^3} \sim \lim_{L\to\infty} \frac{1}{L^2} = 0$$

The integrals will be evaluated by computer and the cross-sections and neutrino luminosities computed.

FOOTNOTES

1. L.F. Landovitz, Nuovo Cimento, 37, 133 (1965).

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2. M. Gell-Mann, Phys. Rev. Letters, 6, 70 (1961)

3. S.G. Matinyan and N.N. Tsilosani, J.E.T.P., 14, 1195 (1962)